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The Various Meanings of Uncertainty

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Abstract

Uncertainty is interpreted differently by different people and disciplines. It can include stochastic uncertainties (i.e. physical randomness), epistemological uncertainties (lack of scientific knowledge), endpoint uncertainties (when the required endpoint is ill-defined), judgemental uncertainties (e.g. setting of parameter values in codes), computational uncertainties (i.e. inaccurate calculations), and modelling errors (i.e. however good the model is, it will not fit the real world perfectly). There are further uncertainties that relate to ambiguities (ill-defined meaning) and partially formed value judgements; and then there are social and ethical uncertainties (i.e. how expert recommendations are formulated and implemented in society, and what their ethical implications are). Some uncertainties may be deep; i.e. within the time and data available to support the emergency management process, there is little chance of getting agreement on their evaluation or quantification.

A key objective of the CONFIDENCE project is to address the decision-making uncertainties in a nuclear accident, but we have not been entirely clear in the proposal of which uncertainties will be considered and how these will be addressed. On the latter point we note that there are several methodologies proposed in the literature for modelling and analysing each type of uncertainty. Sadly not all these are mutually compatible. Unless as a project we make sensible choices of the methodologies to use, we risk producing incoherent, possibly meaningless results and ones that will not stand scrutiny of a post-accident investigation.

The purpose of this paper is to summarise the issues, raise topics for discussion and, where possible, propose ways forward.

Introduction

Few would disagree that uncertainties pervade emergency management and the case of nuclear accidents is no different. Yet RODOS, ARGOS and other decision support systems used by emergency managers pay at best lip service to uncertainty in providing information on the current and predicted situation to the emergency managers and their analysts. Indeed, few procedures followed by emergency managers in their discussions really address uncertainty (French et al., 2017; French et al., 2016). One of the key objectives of the CONFIDENCE project is to consider uncertainty handling more carefully, modifying RODOS and making recommendations on better procedures to take into account the myriad of uncertainties, including social and ethical aspects. However, before CONFIDENCE can do this effectively, there is a need to be clear on what we mean by uncertainties and delineate the aspects addressed in the project. We should recognise that in emergency planning, response and recovery there would be many things on which we will lack knowledge, many issues on which we will be unclear, and that decision support systems and processes may need to support the managers in dealing with these in many different ways. Furthermore, we would expect the support to be consistent in that good decision making needs uncertainties of the same type to be dealt with consistently. Post-accident audit and inquiry would pillorise emergency managers if their actions were driven by incoherence and inconsistency. More importantly, we know that inconsistencies in the management of the Chernobyl Accident led to increased public stress, morbidity and mortality (IAEA, 2006; Karaoglou et al., 1996; Rahu, 2003), and the same seems to have arisen in the aftermath of the Fukushima Daiichi Disaster (Blandford and Sagan, 2016; Hasegawa et al., 2016; IAEA, 2015). Failure to address the overall risk and uncertainty in a coherent, transparent and socially relevant way may – and the evidence shows probably will – impact detrimentally on health and social wellbeing.

Uncertainty is a portmanteau word with many meanings. It relates to being unable to answer questions precisely: e.g.,

- What is the source term, its composition and strength and how will these vary over time?
- How will the public respond in terms of self-evacuation, uptake of stable iodine tablets, and generally following advice and thus conforming to the basis on which protective measures are justified and adopted?

These are just two examples of the many uncertainties that emergency managers and their analysts must consider. Note particularly that neither question relates to a single source of uncertainty, but rather several confounding uncertainties, which need to be tackled consistently before decisions are made. Both examples relate to an inherent lack of knowledge about how things will develop. Both also relate to lack of complete data about the past or the present. In the first case, we may not know precisely the inventory and radionuclide composition of the core before any release, we may not know the energy of the release nor its start time; indeed, we may not know its precise location. In the case of the public, we are unlikely to know who was in the area of the release, whether they followed instructions on sheltering or uptake of stable iodine, the precise protection offered by sheltering in their homes, and so forth.

There are also questions that seem to relate to uncertainty, but which are of a very different type: e.g., the emergency managers may be unclear on their trade-off between immediate public health and long-term public health. Such questions relate to the difficulty of making value judgements and are of a different quality due to a lack of knowledge about external events. Different methodologies and forms of analysis may – we would argue, *will* – be needed to answer them.

With this motivation, the purpose of this paper is to summarise issues relating to uncertainty, raise topics for discussion and, where possible, propose ways forward. In the next section we list several types of uncertainty, not quite in the abstract, but without setting them firmly in the context of nuclear accidents. In doing so we provide many pointers and guides to the literature; it is important to realise that uncertainty is a complex subject and that much work has been done to establish sound, coherent ways of addressing it. Sadly, there are also many naïve papers that offer 'snake-oil' simplistic solutions, falsely promising to deal with complex uncertainties with little thought or effort. We seek to guide the reader away from those. The third section considers specific issues in relation to emergency management and identifies relevant approaches to their resolution. In particular, we focus on two areas important to the CONFIDENCE Project: the threat and early release phase of an accident and the engagement of stakeholders in planning and recovery. At the end of this note we provide a short conclusion and summary table.

Types of Uncertainty: an Overview

Just under a century ago, Knight (1921) distinguished two types of uncertainty: unquantifiable versus quantifiable. The former he and later workers referred to as *strict uncertainty*, the latter as *risk*. Subsequent discussions developed subcategories of these, and Berkeley and Humphreys (1982) discussed seven types of uncertainty. French (1995) discussed ten types of uncertainty and now admits that he forgot some. Taking a perspective from the growth of knowledge, and, thus as a corollary, the reduction of uncertainty, Snowden (2002) recognised four broad categories (see also French, 2013). We could go on. In short, there are many ways of categorising uncertainty and no real agreement on how to do so. So here we will be pragmatic. We make no claim that the list below is exhaustive nor that it separates different types of uncertainty unambiguously. We do believe, though, that it forms the basis for a discussion that we must have urgently within the CONFIDENCE project. Moreover, to emphasise one of the points that we shall develop below: categorising an uncertainty is only a preliminary step towards the more important question of how we should support decision makers in recognising and dealing with that uncertainty in their deliberations.

Stochastic Uncertainties

Many of the most common uncertainties relate to physical randomness. Referred to as *stochastic* or *aleatory* uncertainties, they arise from the randomness within many physical behaviours, from the toss of a coin to the amount of rain that falls within a particular hour at a particular point. Whether the world is truly random or whether it is so complex that the slightest variation in conditions can dramatically affect the outcome of a deterministic behaviour does not matter to our discussion here. What matters is that we cannot predict an outcome with certainty: we need probability. There is general agreement across the scientific and lay communities that probability models are the appropriate means of describing uncertain behaviours in physical systems. School mathematics introduces us to probability in games of chance; and the same theory and principles may be used to analyse and forecast random stochastic behaviours such as share values, the paths of hurricanes, or the spread of a disease. Measurement error is an important application of probability theory and one that is central to defining the likelihood function which lies at the heart of the majority of statistical theory, be it Bayesian or non-Bayesian (Barnett, 1999; French and Rios Insua, 2000; Migon and Gamerman, 1999). The interpretation of probabilities in terms of whether they represent long run relative frequencies (Von Mises, 1957), propensities to adopt different states (Popper, 1959) or a subjective degrees of belief in different outcomes (De Finetti, 1974; 1975) may be moot, but its mathematical use to represent, model and analyse stochastic uncertainties is effectively universal.

Epistemological Uncertainties

Such uncertainties relate to our lack of knowledge. We may be uncertain when something happened, but the time it actually happened is fixed: there is no randomness. In a more scientific context, we may have a number of competing theories to describe some physical behaviour, but we may not know – be *uncertain* about – which, if any, of those theories is true. Epistemology is the study of knowledge and its growth, particularly justified or validated beliefs, so it is natural to refer to such uncertainties as *epistemological*. It is also natural that statistical theory which articulates the process of scientific inference or induction has considered how epistemological uncertainty should be introduced and dealt with in analyses. Frequentist approaches, which once dominated statistical methods, eschew *full* quantification of epistemological uncertainty leaving the scientist to learn intuitively from the evidence displayed to them in the analyses through *p* values, confidence intervals and significance levels (Barnett, 1999). Bayesian approaches, based on quantifying epistemological uncertainty through *subjective* probabilities (see e.g., French and Rios Insua, 2000; Gelman et al., 2013; Savage, 1972) or *logical* probabilities (see e.g., Jeffreys, 1961), now dominate statistical thinking; perhaps more because of the computability of their methodology through MCMC (Markov Chain Monte Carlo) (Gamerman and Lopes, 2006) rather than any great philosophical victory. These approaches, based on Bayes Theorem to formalise rational scientific inference, provide a coherent foundation to statistics as well as machine learning, decision modelling and artificial intelligence (French and Rios Insua, 2000; Korb and Nicholson, 2004; Rogers and Girolami, 2015; Smith, 2010). The growth and success of Bayesian methods over the last half century provide an empirical confirmation that epistemological uncertainties can be handled practically and effectively through probability.

Although once linked to propositional logic and the encoding of knowledge in language, the probabilities used to model epistemological uncertainty are nowadays usually interpreted as subjective degrees of belief (Barnett, 1999; French and Rios Insua, 2000). When used in decision analysis for industry and business as well as individuals, the interpretation may be truly subjective reflecting an individual's or small group's personal beliefs. However, some contexts such as science, government, regulation and emergency management require auditable, open analysis representing something close to objectivity. In these, the interpretation is somewhat different. Probability is taken as representing the uncertainty that an idealised rational person beginning with an agreed body of knowledge would hold in the light of the available empirical evidence.

We should also note that sensitivity analysis has a role in exploring and assessing the implications of epistemological uncertainty for the support of specific decisions. Essentially if all plausible explanations and models lead to roughly the same predictions of the possible outcomes of potential actions, any epistemological uncertainty will not be significant *for that decision* (French, 2003).

There are two further points that we should make. Firstly, if we do not know the parameters of the probability distribution describing a *stochastic* uncertainty, then that lack of knowledge is an *epistemological* uncertainty. Fortunately since both stochastic and epistemological uncertainties can be modelled by probability distributions that obey the same mathematical laws, this is not an issue in the quantitative analysis. Whatever their interpretation, probability distributions behave in the same way in modelling and calculation.

Secondly, Knightian ideas which suggest that some uncertainties, particularly epistemological ones, may not be quantifiable are currently under discussion again, though now they tend to be referred to as *deep* (French, 2015) or *severe* (Comes et al., 2011) rather than strict uncertainty[1]. Some of this discussion is, to be frank, naïve, returning to long discounted approaches that were debated extensively in the 1950s: see, e.g., Milnor (1954) and French (1986). However, the discussions about deep uncertainty do raise issues that are particularly important for emergency management. There are undoubtedly circumstances in which we know too little to build a probability model of our uncertainty *convincingly in the time available*. There may be little agreement among experts about what is happening nor how to model the behaviour. Several explanations of a phenomenon may be plausible and lead to a wide range of quite distinct predictions. Moreover, relevant data may be sparse. In such cases, we might call our uncertainty *deep*, but that does not mean that it will always be deep nor that *conceptually* it could not be modelled by probability. Rather it suggests that we should gather relevant data, engage in discussion with experts and develop an understanding which we can model and about which we can quantify our uncertainty in probabilistic terms. The problem is that to do this takes time, maybe decades in the case of some of the more fundamental uncertainties in science. In emergency management we have little or no time. So we need a way forward, perhaps scenario-focused approaches (French et al., 2017), which may be thought of as gross, quick and dirty sensitivity explorations. We return to this issue below.

Judgemental Uncertainties

Models and computer codes involve parameters, many of which are set judgementally by the users drawing on their expertise. Some parameter choices may be embedded in the code, barely noticed and set to default values, but those default values will again have been set by judgement, perhaps by the code's creators. In very few cases will these parameters be known precisely; to some extent their values will be best guesses. So the user must consider how the specific choice of these parameters may affect the predictions of the code. In some cases this may be done using Monte Carlo methods (Evans and Olson, 2002), drawing samples from probability distributions representing the parameters' uncertainty, though this risks an infinite regress relating to uncertainty about hyper-parameters in the distributions used. Alternatively a variety of more deterministic sensitivity analyses may be conducted (French, 2003; Saltelli et al., 2000a; Saltelli et al., 2000b; Saltelli et al., 2004).

Computational Uncertainties

We often talk of using a model to describe some behaviour, but more often than not we use a sequence of models (French, 2015). We may begin with a cognitive model which describes our understanding of the behaviour. It may embody complex scientific laws, expressed as mathematical formulae relating inputs precisely to outputs, or it may be a regression model, expressing little more than the correlations between inputs and outputs, but again giving us a mathematical formula. In some cases it may be an implicit model: e.g. the solution of a partial differential equation. In calculating with that model using a computer code we usually make a number of computational choices: e.g., mesh size, number of iterations or convergence criteria. Computer arithmetic is only accurate to a finite number of decimal places and, although many techniques are used to avoid the build-up of numerical errors, they are not perfect. The result is that the user will not know how well the computational code output matches the results that perfect calculation of the original model would produce: i.e. there are computational uncertainties. It may be that the computer code is not tractable in reasonable time, so further approximations may be introduced to increase speed and, inevitably, these increase computational uncertainty. Statistical emulation of computer codes takes

[1] Note that there are further meanings of *deep* uncertainty in the machine learning and artificial intelligence literatures which are not relevant to our discussions here.

this one step further by fitting a complex model with a much simpler Gaussian process, a sort of functional regression (Conti et al., 2009; Craig et al., 2001; Goldstein, 2011; O'Hagan, 2006). Apart from the approximations brought by emulation itself, the algorithms used to emulate a complex computer code themselves involve choices of sampling points, convergence criteria, etc. that contribute to the overall computational uncertainty. In the past, much effort has been expended in producing bounds on computational errors in specific calculations. Emulation algorithms provide some assessment of their own accuracy. Recently Hennig et al. (2015) have promoted probabilistic techniques for representing overall computational uncertainty, following earlier work by, e.g., O'Hagan (1992). The point we need to note here is that computation increases the overall uncertainty in the numerical result.

Model Uncertainty

However good the model is and however good our computations, it will not fit the real world perfectly. Even if there were no computational approximations used in its calculation and even if there were no stochastic elements to the real world behaviour described, the model would not be perfect. As the truism says: the only true model of reality is reality itself. Over the years attempts have been made to model the gap between a model and reality (Blight and Ott, 1975; Brynjarsdóttir and O'Hagan, 2014; Draper, 1995; French, 1978; O'Hagan, 2012), the latest being discussions of reification (Goldstein, 2011; Goldstein and Rougier, 2009). But the task, though informative in understanding the process of modelling, is fruitless, creating an infinite regress of models modelling errors of modelling error models. In many cases, this is a conceptual nicety since the models concerned are clearly more than accurate enough for the task concerned: e.g. calculating a road distance between two points from a map on a GIS. But in other cases, modelling error may be significant and that the model only gives broad indications of the real behaviour: e.g. a model of the spread and migration of an animal population. The papers cited above provide some techniques to allow for modelling error in fitting models to data, broadly inflating variances to smooth the fitting process. However, in using models for prediction one has to rely on the user's experience to allow for 'how good the model is' (Kuhn, 1961).

Ambiguity, Lack of Clarity and Endpoint Uncertainties

In many respects, judgemental, computational and model uncertainties are simply specific cases of epistemological uncertainty: they relate to a lack of knowledge. Ambiguity and lack of clarity are entirely different forms of uncertainty. They relate to our not having defined clearly what we mean by some wording: e.g. the description of a consequence. Some researchers have suggested modelling ambiguity and lack of clarity with fuzzy mathematical concepts (Kacprzyk and Zadrozny, 2010; Yager and Zadeh, 2012), and these methods have had some success in natural language processing. But generally such modelling does not provide the appropriate way forward in supporting decision making (French, 1995). When making a decision, we do not need a model of some ambiguity or lack of clarity, particularly in the description of the strategies, the consequences and value judgements that will drive our choice. Rather we need to think through our position and resolve these by conscious deliberation. A common approach to this is via facilitated workshops in which the facilitator continually challenges participants to explore and define much more clearly what is meant by phrases such as 'minimising health effects' (Eden and Ackermann, 1998; French et al., 2009; O'Brian and Dyson, 2007). Such resolution invariably requires value judgements, most often in the form of trade-offs between the different attributes involved in describing some entity or some objective: e.g. between life expectancy for different ages in describing overall health.

To resolve *endpoint uncertainty* we need to consider how the consequences in a problem should be defined so that the models and analysis consider those aspects of the outcome of an action that are most important in making a decision. What time horizon should be used? For instance, in responding to the immediate threat of a nuclear accident, should we be concerned only with immediate risks or do we consider the risks from contamination that may extend decades or centuries into the future? Do we consider impacts just to health, or should we also consider the environment, the local economy, society and the cost of any action? There are risks in assuming that the key endpoints are so obvious that they do not need to be clearly agreed and stated. For example, without discussion, it should neither be assumed that direct health-related impact is the only significant endpoint, nor that the outputs from current coding tools are the only results to be considered in decision-making. Decision makers may feel uncertain about the answers to these questions. But, like ambiguity and lack of clarity, this uncertainty is of a very different nature to stochastic and epistemological uncertainties. Endpoint uncertainty relates to being comfortable with the depth and detail of modelling. In Phillips (1984) terms, it relates to deciding whether a model is 'good enough' or sufficiently *requisite* to support a decision. Endpoint uncertainty, as with ambiguity and lack of clarity, can only be resolved by discussion and deliberation, perhaps through a facilitated workshop.

Endpoint uncertainty is not just important in itself, it has implications for what other uncertainties need be modelled and analysed and to what depth. Once we know what we are trying to assess and what is really important to us, we can ignore uncertainties that do not feed through to these endpoints.

Social and ethical uncertainties

Many uncertainties relate to value judgements. The emergency managers and those in charge of recovery need to consider how to balance different types of cost relating to strategies and their impacts: health, social, environmental, economic, etc. For instance, managers may be charged with *minimising health effects*, but may not know precisely what is meant by this. What is a health effect? The imperative to *minimise* implies that they must be quantified in some way. But in what way? By number, scale, some combination? Does it matter who suffers the health effect? Should they care more about health effects in children than adults? If the risk is long term, is the focus on immediate or long term health effects in present populations or the health of future generations? Is a physical health detriment to a few more important than a mental health detriment to many? There are a host of uncertainties which need to be unpacked and defined before the imperative to 'minimise health effects' can be operationalised and followed. These uncertainties relating to values and ethics clearly have a different character compared to stochastic or epistemological uncertainties.

Moreover, in resolving such uncertainties, we should recognise that decision makers often aim at representing a wider group of stakeholders, maybe an organisation, a local community or the wider public. This brings to the fore the question of whose values and ethics should be drawn into the decision making. The decision makers need to understand and articulate the values and ethics of the people whom they represent. This can bring into the mix some epistemological uncertainty in which the decision makers seek to learn what their constituents want. Methods of opinion polling may be used which can result in formal probabilistic representations of public values in some sense. But in complex cases, stakeholder workshops and other interactive forms of engagement are to be used to provide the decision makers with a qualitative understanding of the values and ethics that should flow through their decisions.

Experience, notably from the Chernobyl and Fukushima accidents but also from non-nuclear accidents, shows that stakeholders' values, ethical considerations, requirements for public communication and the contrasting needs and concerns of people in different environments are key factors influencing the effectiveness of risk assessment and management. In particular, the inherent social uncertainties, the different perceptions of risk, and the societal (dis)trust issues pose important challenges to radiological risk governance. Social and ethical uncertainties are most often used to describe the way recommendations and information is taken up by lay people and other publics (i.e., whether the advice given by modellers and/or authorities is acted upon). Models are always based on assumptions about the social context where decisions take place (e.g. that people will accept to live in contaminated territories). Therefore, the efficiency of protection strategies depends significantly on the way the social context is understood and accounted for in decision-making. Social and ethical uncertainties can also be attached to the decisions, choices and assumptions made by modellers, scientists and other experts during their 'scientific' assessment (i.e., the selection of data, coefficients, criteria, target populations or reference organisms, levels of significance for statistical testing, etc.).

Social uncertainties in how expert recommendations are implemented in society may refer to public acceptance and compliance with protective actions advice; social and economic consequences of the recommendation and actions, and uncertainties in those consequences; and the level of stakeholder and public engagement used or planned.

Ethical uncertainties may refer to: e.g.

- defining the level at which a risk becomes acceptable (e.g. 10^{-7} for the annual risk to an individual;
- whether members of a population feel that they have given consent to being exposed to a particular level of risk;
- being sensitive to inequalities in the distribution of risk;
- any mention of the way in which autonomy, governance, responsibility, transparency might impact on public acceptance of risk.

Social and ethical uncertainties can also be recognized in expert recommendations. For instance, is there any discussion on possible societal or economic consequences? Are challenges of criteria selection (e.g. worst case? vs best possible estimate) discussed? And are any of the above taken up in the expert/authority recommendations or decisions?

Cynefin, Epistemology and Uncertainty

We have described several different types of uncertainty and noted that they need be addressed in different ways. We should also note that the surrounding context and our knowledge of other aspects also shape how we should deliberate on, model or analyse uncertainty. Snowden (2002) introduced the Cynefin framework to do this. French (2013) discusses Cynefin in relation to decision support, and French and Niculae (2005) use it to explore aspects of emergency management (see also Niculae, 2005).

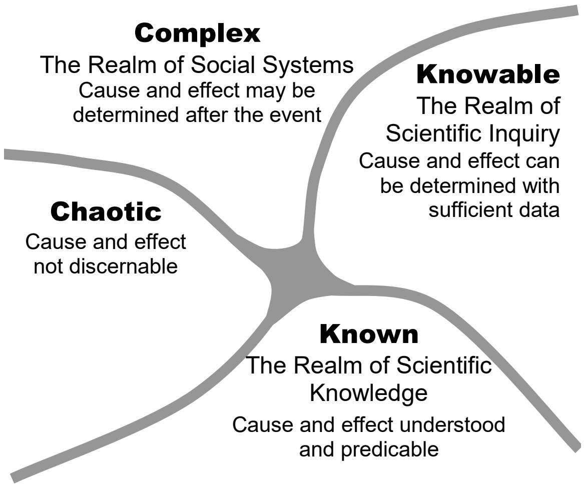

Figure 1: Cynefin

Snowden's Cynefin model roughly categorises decision contexts under four headings: see Figure 1. In the *Known Space* – also called the *Simple Space* or, more informatively, the *Realm of Scientific Knowledge* – the relationships between cause and effect are well understood. All systems and behaviours can be fully modelled. In the *Knowable Space* – also known as the *Complicated Space* or, again more informatively, the *Realm of Scientific Inquiry* – cause and effect relationships are generally understood, but there is a need to gather and analyse further data to set parameters in models before any predictions can be made. In the *Complex Space* – also called the *Realm of Social Systems* – knowledge is at most qualitative; too many potential interactions exist to disentangle particular causes and effects. There are no precise quantitative models to predict system behaviours such as in the Known and Knowable spaces. This is often the case in many social systems, though such complexity can arise in environmental, biological and other contexts. Finally, in the *Chaotic Space* there are no obvious candidates for cause and effect. We simply do not know what is happening and have yet to make sense of things. Modelling and quantitative analysis are impossible because we have no concepts of how to separate entities and predict their interactions. Deep uncertainties, by and large, arise in the Chaotic and Complex Spaces. In the Knowable and Known Spaces, it is usually possible to model stochastic and epistemological uncertainties probabilistically, and perhaps use sensitivity analysis to assess the implications of judgemental and computational uncertainties.

Moving from the Chaotic Space through the Complex and Knowable Spaces to the Knowable Space, our knowledge and understanding move from very deep uncertainty to certainty. Epistemology from sense-making through inference to full knowledge can be described very simply against the backdrop of Cynefin (French, 2013). We would also note that our knowledge of our values change as we move through the spaces. In the Known and Knowable Spaces, familiarity with many similar situations means that we will have thought through our values previously. We know what we want to achieve simply because we 'have been here before'. Such is not the case in the complex or chaotic spaces. Novel issues and lack of full understanding require us to reflect upon what we want to achieve (Slovic, 1995). With CONFIDENCE, we will need to work with stakeholders to help them deliberate on what their values are, contextualising their fundamental values to the circumstances that they face.

The Uncertainties that CONFIDENCE needs address

The strapline of the CONFIDENCE Project is: *coping with uncertainty for improved modelling and decision making in nuclear emergencies.* Its work-programme expands on this, seeking "to understand, reduce and cope with the uncertainty of meteorological and radiological data and their further propagation in decision support systems, including atmospheric dispersion, dose estimation, food-chain modelling and countermeasure simulation models. Consideration of social, ethical and communication aspects related to uncertainties is a key aspect of the project activities." Thus it is clear that understanding, modelling, management and communication of uncertainty is central to the objectives of CONFIDENCE. In this section, we discuss how the different types of uncertainty discussed above will enter into this. We exemplify the approach proposed by focusing on two specific areas:

- the threat and early release phase of an accident, including source term, atmospheric dispersion and deposition, and health impact modelling;
- accounting for stakeholders' preferences in planning for and recovery after a nuclear accident.

Our approach is simplified to focus on key principles of dealing with uncertainty. Thus we do not explicitly address hydrological dispersal, agricultural production and food-chain modelling, as well as economic and environmental impacts. Though these processes also are subject to many uncertainties, they do not introduce further conceptual issues in uncertainty modelling.

Uncertainties in the threat and release phases

Figure 2 presents, in a highly simplified format, the major models that contribute to estimating health impacts during the threat and early phases of a nuclear accident. Its arrows should be read as showing information flows and not temporal relationships. Many models are iterative, as is the entire modelling network. Thus to estimate atmospheric dispersion and deposition, estimates of the source term and local weather forecasts will be required. In turn, if we assume the imposition of swift agricultural controls, health impacts will result primarily from environmental contamination given by the atmospheric dispersion and deposition outputs. All these calculations will draw upon topographical, population and other spatially referenced data collected by a variety of measurement techniques and then stored and perhaps interpolated ('kriged') in a Geographic Information System[2] (GIS) and on assumptions about public behaviour and adoption of advice on countermeasures. Related discussions of the modelling and inherent uncertainty producing radiological emergency response assessments may be found in Haywood (2010) and Haywood et al. (2010).

[2] A Geographic Information System (GIS) is defined by some as little more than a spatially referenced data base; others emphasise the system aspect and take GIS to include all the statistical, analytic and visualisation algorithms used to interpolate and present data in querying and using a GIS. We adopt the latter.

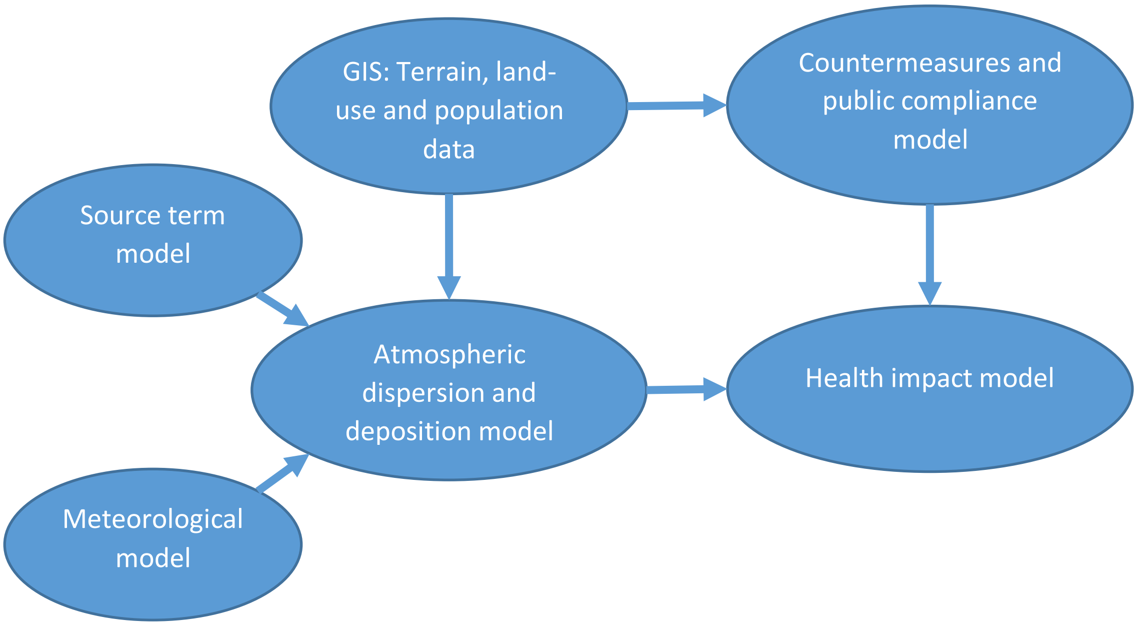

Figure 2: A simplified presentation of the different models that contribute to predicting the health impacts during the threat and release phases of a nuclear accident.

When an accident threatens and during any release, the *source term* is hugely uncertain in many respects: its composition, its profile, its duration, its energy (heat), etc. Some attempts have been made to quantify these uncertainties into broad brush categories using belief nets (Grindon and Kinniburgh, 2004), but broadly the approach has been to produce point predictions based on engineering and nuclear thermodynamic models or simply rely on expertise. It is worth noting that if expertise is to be used, there seem to be no plans to use structured elicitation and avoid biases that may be present when experts are asked for assessments without any formal protocol (Dias et al., 2017; O'Hagan et al., 2006). Source term uncertainties are certainly epistemological and probably contain some stochastic elements too. The models used will introduce computational and probably judgemental uncertainties too as some parameters will be set by expertise. However, *conceptually* the overall uncertainty from these aspects can be modelled by probability distributions. Unfortunately, the uncertainties are also likely to be deep: i.e. it is unlikely that the probability models can be produced quickly and convincingly enough within the time needed by emergency managers. We return to this point below.

Weather forecasting is regularly offered as an example uncertainty that we all experience, and the meteorological models here will be no exception. Even though meteorological offices use some of the highest power computing available, their predictions of wind and precipitation will be subject to stochastic, epistemological, judgemental and computational uncertainties. All can be modelled probabilistically and, indeed, meteorology is one of the sciences to adopt probability modelling as fully as is currently possible. The majority of these uncertainties will not be deep, though if there is a possibility of a front passing through the region, its timing may be highly uncertain. The timing and location of showers may be even more uncertain.

There are many atmospheric dispersion and deposition models, each with its own characteristics and set of approximations. Choosing a puff or particle model, frequency of puffs or number of particles, grid size, etc. all introduce judgemental uncertainties. Moreover, these models take outputs from source term[3] and weather models, and from terrain data from the GIS, adding to the uncertainty. The algorithms used to run the models introduce computational uncertainties, and there are two further issues that relate to uncertainty. First, some analyses use ensemble[4] techniques, running the models with a sample of different initial conditions, particle releases and other inputs. There is a common assumption that the set of ensembles produced provide a representation of the uncertainty in the dispersion and deposition predictions. This may not be the case for a number of reasons. The sample of initial conditions, particle releases, etc. may not reflect the actual uncertainties on the inputs to the model, being sampled from uniform distributions rather than the uncertainties coming from the source term and weather modelling and the GIS. Moreover there is no attempt to introduce the judgemental uncertainties from model choice, parameter setting, etc. The second issue concerns the judgemental uncertainty on seemingly the same parameters in different models. The release height of the source term is an input to many of these models. It gives the *notional* height at which the plume stops rising and begins to spread out. Given possible wind shear, this parameter has a significant effect on determining the direction in which the plume moves off. Analysis against data sets from experimental releases has shown that the numerical value of the release height that gives the best fit for one dispersion model may be quite different to that gives the best fit for another model. In other words, the judgemental uncertainty on a parameter of the source term depends on the dispersion model being used. In general, this issue is true whenever the output of one model is taken as the input for another, but it is particularly apparent in this case. Finally, the interdependence of the model runs in the ensemble needs to be considered. Since they use the same model code they are not statistically independent and thus the output of the ensemble runs will not represent the full uncertainty in the dispersion and deposition predictions.

Emergency decision support systems such as ARGOS and RODOS rely on spatially referenced data from a GIS: residents, industries and businesses including number of employees, dwellings including form of construction, terrain and topography, land use including agricultural production, schools, hospitals, and so on. This will introduce further uncertainties. Firstly, the data itself will be subject to error: even if accurate when input, people move, land use changes, etc. Secondly, the granularity of the data in a GIS can have quite a gross character. For instance, land use may be recorded as constant over a 100m grid square, so the data extracted from a GIS may be produced by kriging to produce approximate point values at grid points. The temporal variability of such datasets is a major source of uncertainty, for example whether the dataset represents a daytime or night-time population, and the impact of seasonality (e.g. tourism) and daytime variations (e.g. population movements due to school and work). Not least is the effect of the emergency itself on the accuracy of the data used in the assessment, e.g. unplanned and spontaneous evacuation movements.

The final two models in Figure 2 are of a different character to those that provide predictions of atmospheric dispersion and deposition. There is no detailed modelling: human behaviour is not so well understood. Rather several gross behavioural assumptions are made to get what are effectively ball park figures. Simple multipliers are introduced to reduce the effectiveness of a countermeasure: e.g. given advice from the emergency managers to take stable iodine, only ξ% will successfully do so or η% will self-evacuate despite advice to shelter. This means that these models introduce very significant judgemental uncertainty, and also judgements that will be more applicable to some emergencies than to others, or to some areas than others. Moreover, in using the linear no-

3 Price et al (2017) have recently surveyed the sensitivity of atmospheric dispersion deposition models to parameter settings in six source term models.

4 Meteorologists use the term *ensemble* in similar but subtly different ways to statisticians. Here we mean multiple runs of the same model code with different plausible initial conditions and parameters.

threshold hypothesis to estimate cancer risks further gross assumptions are made about the distribution of people across the contaminated environment (Argyris and French, 2017).

We have omitted to discuss modelling error in our discussion here. There are two reasons for this. Firstly, while we could consider modelling errors at each of the nodes in Figure 2, it perhaps makes more sense to consider the overall modelling error taken together in so far as it predicts health impacts with and without countermeasures. The intermediate errors may not be as relevant in supporting the emergency managers. Secondly, our remarks in the previous paragraph about the gross behavioural assumptions regarding compliance and the effectiveness of countermeasures raise the question: how accurate do those elements contributing to the predictions of atmospheric dispersion and deposition need to be? We return to this point below.

It is clear from the above that a wide range of stochastic, epistemological, computational, judgemental and modelling uncertainties are involved in producing predictions of health impacts to provide the emergency managers with information and guidance on their decisions. So how are these currently presented to emergency management teams? Hardly at all! Point estimates are provided and uncertainty is assumed to be handled by discussion between the emergency managers and their support teams (French et al., 2017; French et al., 2016). Those discussions usually relate to stochastic and epistemological uncertainties with judgemental, computational and modelling uncertainty seldom mentioned. It is well known that scientists typically underestimate errors in their model predictions, possibly because the models give plausibility to their results and plausibility is at the heart of a psychological bias decreasing uncertainty (Kahneman, 2011; Selin, 2014). So it is likely that these discussions do not provide emergency managers with a full appreciation of the overall uncertainty in the predictions being offered to them.

There have long been intentions that decision support systems such as RODOS should provide more formal treatments and assessments of uncertainty for the emergency managers (Caminada et al., 2000; French, 1997); and there have been efforts to use Kalman filtering in atmospheric dispersion and deposition models not only to provide uncertainty assessments but also to perform data assimilation (Politis and Robertson, 2004; Smith and French, 1993). However, these have not been sufficiently developed to be implemented and, moreover, are very limited in the uncertainties included in the modelling. No real modelling of the source term uncertainties is included, nor of terrain uncertainties, and there is no subsequent modelling of the uncertainties inherent in forecasting health effects. Clearly to develop RODOS modules which would include assessments of all the uncertainties mentioned above during the CONFIDENCE project would be a major undertaking, and beyond the scope and timescale of this work. Moreover, we should remember that that some of the source term uncertainties may be deep.

One approach that we might investigate would be to provide emergency managers with several scenarios. Scenario analysis is used throughout business and government to develop strategic thinking (Schoemaker, 1995; van der Heijden, 1996) and to challenge too great a focus on one specific prediction. The most basic forms of scenario analysis develop a series of maybe 4 or 5 scenarios that are 'interesting' in some sense and may be used as backdrops for discussion about the merits and risks of different strategies. How 'interesting' is defined is moot, with many possibilities. In the case of the threat and early phase of a nuclear accident, we might consider:

- reasonable best and worst cases of some form – useful for bounding possibilities;
- a likely case – useful for maintaining a balanced perspective;

- an assumption that a particular event happens or does not – useful if a key event, such as a second release or the arrival of a weather front, is unpredictable and shrouded in deep uncertainty.

Note that several reasonable best, worst or likely cases might be explored and considered, since no single case will illustrate all potential impacts. Note also that to select a small set of scenarios for further analysis and discussion will inevitably mean that several more, perhaps many more will need be generated and examined quickly. However, only a handful of scenarios would be developed fully and shown to the emergency managers. In crisis management, there is no time to do more. There is also the issue of cognitive capacity in that decision-makers often cannot absorb and balance out the implications of many scenarios (Miller, 1956). French et al. (2016) survey the relevant literature on developing an appropriate set of scenarios for the emergency managers to consider.

The presentation of each scenario would include maps or sequences of maps showing the evolution of events under the assumptions implicit in its definition: for an example see Figure 3. The design of RODOS, ARGOS and other DSS also allow more dynamic presentation of scenarios which would present the emergency managers with several evolutions of the plumes and corresponding regions in which protective measures might or would be needed under national and international guidance. Some initial work has been undertaken to explore these ideas (Comes et al., 2013; Comes et al., 2015; French et al., 2016; French and Bayley, 2003; Havskov Sørensen et al., 2014; Haywood, 2010; Raskob et al., 2009). It is important to realise that the scenarios are neither mutually exclusive nor span/partition the future, so assigning probabilities to them is meaningless. The key idea in presenting several scenarios is to stretch the crisis managers' thinking and make them consider a wide range of possibilities. It is important, of course, to guard against framing and plausibility biases. This might be done by a continual process of challenge to justify their thinking implicitly (French et al., 2009).

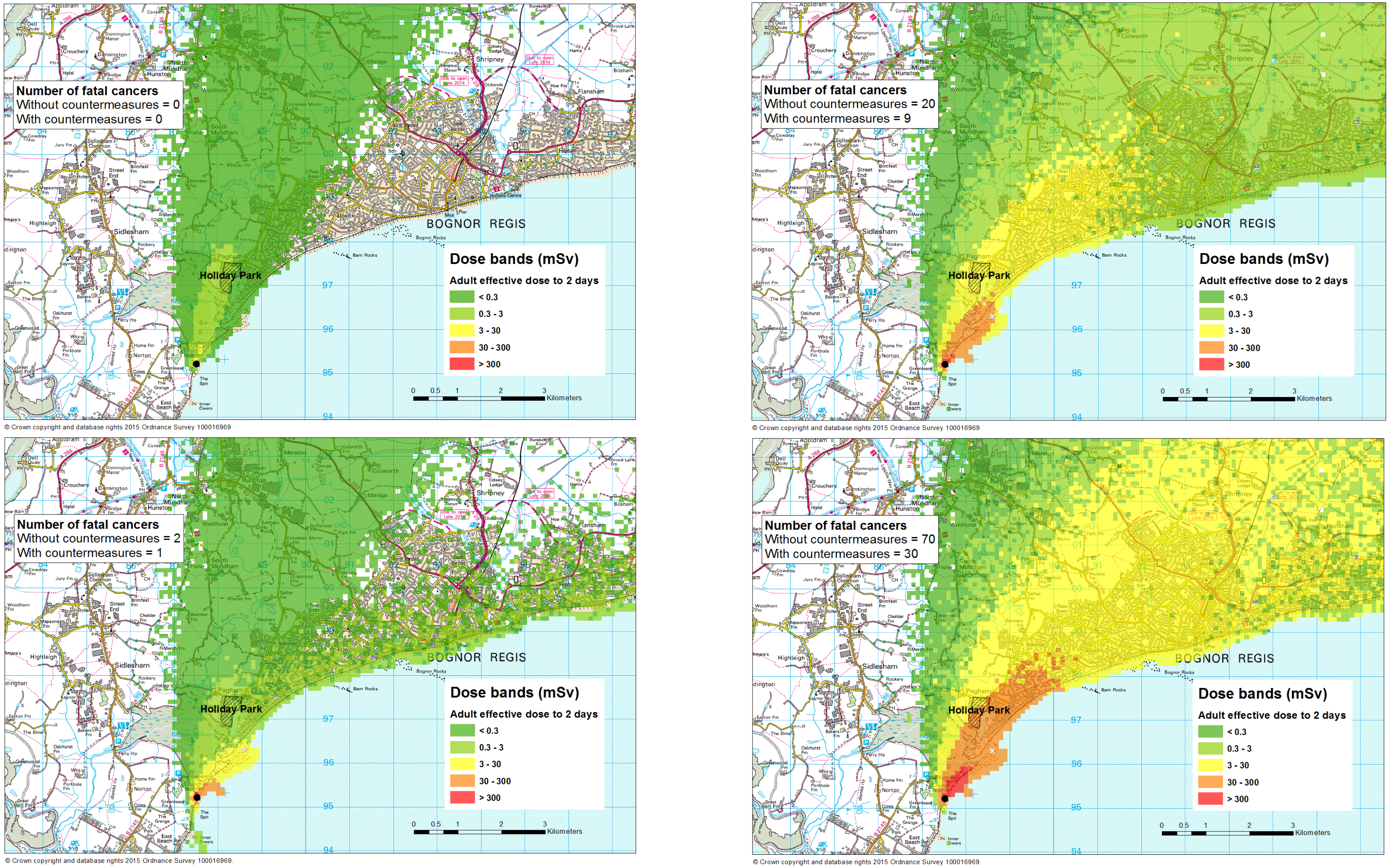

Figure 3: Four scenarios used in a UK workshop to explore alternatives to a single ‘reasonable worst case’ (RWC).

Uncertainty and Stakeholder Engagement

During the urgency of the threat and release phases, it would not be possible to consult stakeholders and the local public in any meaningful fashion, therefore assessments of public behaviour should be made in the preparedness phase, based on experience from past accidents, incidents and exercises, and modelling of expected behaviour. Moreover, the public should be provided with the relevant information (information needed for their own decision-making) and guidance as soon as possible; and note that this guidance should often include assessments of uncertainty. In some countries stakeholders and the local public are regularly consulted during the planning for potential nuclear accidents and the intention is that in the event of a future accident, clean-up and recovery strategies would be discussed with them in depth. In doing this, there will be a need to explore many uncertainties with them. Work Packages 4 – 6 of the CONFIDENCE project are specifically focused on these issues. Within these work packages we need to consider three broad aspects of uncertainty.

i. What is lay persons' and emergency actors' understanding and processing of uncertain information and their subsequent behaviour in nuclear emergency situations;
ii. What are the main sources of social and ethical uncertainties in emergency situations and the transition phase and how to reduce these through better communication, according to the information needs for each particular stakeholder group
iii. how to learn *from* the stakeholders and the public their preferences on clean-up and recovery strategies and integrate them into decision-making, recognising that they may be unclear on their valuation of these.

In relation to point i we observe that while perception of risks from nuclear accidents and radiological contaminations of the environment have been extensively investigated in the literature, very few empirical studies have focused on lay public behaviour in nuclear emergency situations. However, a substantial body of research, mostly grounded on social psychology, exists regarding lay public preparedness for natural hazards such as flood, earthquakes or hurricanes. Case studies of past accidents and incidents, mental models approaches, naturalistic observation can provide additional insights.

On point ii there has been a substantial volume of research and guidance published since the earliest post-Chernobyl projects (Drottz-Sjöberg and Sjoberg, 1990; Havenaar et al., 2003) and more widely in other domains (see, e.g., Bennett et al., 2010; Fischhoff, 1995; OECD, 2002; Renn, 1998; US DHHS, 2002). Reviews of risk perception and risk communications literature can be found in (Renn, 2008). More research is needed on communication of uncertainties. In particular, French et al. (2016) note the dearth of research and guidance on also communicating spatial risk and presenting uncertainty on maps.

It is point iii on which we shall concentrate here: how do we learn from stakeholders the values that they should like to drive emergency and recovery decision making. Firstly, we should note that stakeholders may be unclear on their values in relation to emergency response to a nuclear accident. Thankfully, the vast majority of people have not experienced a serious threat of release of radiation from a nearby nuclear plant. For them, it would be an entirely novel situation and thus it would be categorised as lying in Cynefin's Complex Space. As we noted, people seldom have clearly formed values in relation to such situations: they will still be learning and thinking about what the experience means for them. In the event that an accident has occurred and they are being consulted on recovery, this may be particularly true: a catastrophe and its aftermath can change people's fundamental values (French et al., 1997). Thus we cannot simply ask stakeholders for their values. We need to help them discuss, think about and, indeed, form their values and preferences. Many of the approaches to stakeholder engagement and public participation in decision making use multi-criteria decision analysis (MCDA) to articulate such exploratory discussions (Gregory et al., 2013; Papamichail and French, 2013; Rios Insua and French, 2010). CONFIDENCE work packages 4 – 6 will be trialling such methods, taking stock of work from earlier projects such as EVATECH and EURONOS.

Like 'uncertainty', MCDA is a portmanteau term covering many methods and approaches, sadly often incompatible approaches (Belton and Stewart, 2002; Bouyssou et al., 2000; Bouyssou et al., 2006). There are several schools of MCDA, each based on its own set of assumptions, sometimes explicitly stated, sometimes left implicit. Multi-attribute value theory (MAVT) provides arguably the most justified and frequently used approach being based upon explicit, well discussed and explored sets of assumptions (Keeney, 1992; Keeney and Raiffa, 1976; Krantz et al., 1971; Wakker, 2013). There are linear and non-linear versions appropriate to different sets of attribute preferential independence conditions. MAVT approaches may be developed naturally into expected utility models and are entirely compatible with Bayesian methods of inference and decision (French and Rios Insua, 2000; Keeney and Raiffa, 1976). These methods have been used in many applications and are implemented in many software packages, including RODOS (Bertsch et al., 2009). The Analytical Hierarchy Process (AHP) is another MCDA approach, based on less well explored assumptions (Saaty, 1977; Saaty, 1980). Critics point to certain paradoxical behaviours known as rank reversal (Belton and Gear, 1983; Saaty and Vargas, 1984); but others have noted that if used to assess weights only and not marginal value functions, such behaviours are not an issue (Salo and Hämäläinen, 1997). When steps are taken to counter rank reversal, AHP and MAVT approaches are reasonably compatible. An entirely different set of approaches are based on outranking ideas (Roy, 1996; Roy and Vanderpooten, 1996). These lead to methods such as the ELECTRE family of approaches (Roy, 1990) and PROMETHEE (Brans and Mareschal, 2005). Again these methods have found many applications (Roy et al., 1993).

Outranking approaches are quite different to MAVT and AHP ones. They allow incomparability between options, non-compensatory approaches and the use of pseudo-criteria (implementing the concepts of indifference, weak preference and strict preference). They use some of the same technical terms with subtly but significantly different meanings. This can lead to confusion among stakeholders and, unfortunately, many analysts. For instance, all MCDA methods introduce 'weights' which measure the relative importance of different attributes *within the structure of their models*. So it is possible that two distinct MCDA methods might assign 0.7 weight to an attribute such as health impacts arising from radiation exposures. However, although the numerical value 0.7 is the same in both cases, the relative importance represented might be different. Weights of attributes cannot be compared simply across different MCDA models (Gershon, 1984; Steele et al., 2009). This means that great care must be used when setting up a series of stakeholder engagements using MCDA methods to ensure that the methods employed use compatible concepts. Otherwise, far from becoming clearer on stakeholder values, we may in reducing one set of uncertainties introduce several others.

Returning briefly to point iii above, a CONFIDENCE note is being prepared by Tim Müller on presenting uncertainties within MCDA.

Using MCDA in an engagement workshop or, perhaps, interactively on the web will allow us to learn about certain stakeholders' values: in the first case, those of the attendees, in the second those of visitors to the site who are able to comprehend and use the MCDA model provided. Neither case will summarise the opinions of the entire population of stakeholders unless some care is taken to consider and ensure the representativeness of the participants. Representativeness, normally thought of as a governance issue, is also important in reducing endpoint uncertainty and lack of clarity on values.

Conclusion

In planning for, managing and recovering from a radiological accident, there are many uncertainties that have to be assessed, analysed and communicated to emergency managers and stakeholders. Some may be modelled by probability, some explored and bounded through sensitivity calculations, and some relating to lack of clarity may be resolved by introspection and discussion; but some may be deep and allow only cursory assessment and analysis in the time available. There is no single methodology that enables analysts to address the myriad of uncertainties facing emergency managers. In CONFIDENCE we will need to draw on many approaches to cope with uncertainty for improved modelling and decision making in nuclear emergencies, and we shall need to ensure that the approaches we use are based on compatible sets of assumptions. Coherence and consistency are important.

Table 1 provides a summary of the different forms of uncertainty and approaches to modelling and analysing them.

Finally, we note that the European Food Safety Authority (EFSA) is currently producing extensive guidance on how to identify, analyse, present and communicate uncertainty. The report is still in draft, but provides much complementary material to this paper which recognizes the imperatives and responsibilities on scientists providing advice to public officials and regulators (EFSA, 2016).

Table 1: Summary of the different forms of uncertainty and approaches to modelling and analysing them

Uncertainty	**Examples**	**Approaches to modelling and analysing**
Stochastic (physical randomness)	• Occurrence and patterns of precipitation • Actual numbers and locations of the local population at the time of the release • Long term radiation related health effects	• Probability modelling and statistical analysis
Epistemological (lack of scientific knowledge)	• Source term characteristics: time profiles of radionuclide mix, energy, etc. • Course and shape of plume and deposition	*Normal uncertainty* • Probability modelling and statistical analysis *Deep uncertainty* • Exploration of several scenarios
Judgemental (e.g. setting of parameter values in codes)	• Parameters within models and computer codes • Compliance of population with advice on protective measures	• Sensitivity analysis • Monte Carlo analyses
Computational (inaccuracy in calculation)	• Accuracy of approximations used in atmospheric dispersion and deposition models	• Bounds from numerical analysis • Probability modelling of error distributions if stochastic approximations or statistical emulation used
Modelling (i.e. however good the model is, it will not fit the real world perfectly)	• Discrepancy between model and reality if model based on accurate parameters and data and calculations performed perfectly	• Experience ….
Ambiguity, Lack of Clarity and Endpoint (ill-defined meaning)	• How should Endpoints be described, what matters • Importance of different attributes in evaluating endpoints	• Stakeholder workshops using facilitation to challenge thinking

Uncertainty	Examples	Approaches to modelling and analysing

Social and ethical (i.e. how expert recommendations are formulated and implemented in society, and what their ethical implications are)	• How expert recommendations are formulated and implemented in society
	  • Acceptance of risk
	  • Ethical issues: risk distribution, autonomy, governance, responsibility, transparency
	  • Communication
		• Social psychology, mental models, naturalistic observation
		• Ethical principles of radiological protection
		• Communication experiments

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